On the energy-shell contributions of the three-particle - three-hole excitations

A. Mariano† and F. Krmpotić†

Departamento de Física, Facultad de Ciencias Exactas,

Universidad Nacional de La Plata, C. C. 67, 1900 La Plata, Argentina

and

A.F.R. de Toledo Piza

Instituto de Física, Universidade de São Paulo C.P. 20516, 01498 São Paulo, Brasil

Abstract

The response functions for the extended second and third random phase approximation are compared. A second order perturbation calculation shows that the first-order amplitude for the direct $3p3h$ excitation from the ground state cancels with those that are engendered by the $1p1h$-$3p3h$ coupling. As a consequence nonvanishing $3p3h$ effects to the $1p1h$ response involve off energy shell renormalization only. On shell $3p3h$ processes are absent.

†Fellow of the CONICET from Argentina.

Many efforts have been devoted during the last few years in developing generalized random phase approximations (RPA), which go beyond the standard one-particle - one-hole ($1p1h$) approach [1]. This has been accomplished by including additional correlation effects in both the ground state and the excited states [2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16]. The reasons for that were mainly: i) the problem of the missing strength in the Gamow-Teller (GT) resonances, induced by (p,n) reactions [17, 18], and ii) the issue of the missing charge and missing dip-strength in quasielastic electron scattering [19]. In particular, the extended second RPA (ESRPA), which explicitly includes the $2p2h$ ground state correlations (GSC), was extensively used to describe the above mentioned nuclear excitations [2, 4, 5, 6, 10, 11, 12, 13, 15]. Yet, it is self evident that when the $2p2h$ admixtures are present in the ground state, the external excitation field can lead, not only to the $1p1h$ and $2p2h$ states in the final nucleus, but also to the $3p3h$ states. However, as the ESRPA does not involve the $3p3h$ propagator these excitations cannot appear within the response function as real on the energy-shell processes. Recently the $3p3h$ degrees of freedom were explicitly included within a Tamm-Dancoff approach (TDA), and their effects on the non-energy-weighted GT sum-rule were discussed [14]. Also an extended third RPA (ETRPA), which possesses as the TDA limit the formalism developed in ref. [14], has been used to study the effects of $3p3h$ excitations on the static strength function for quasielastic electron scattering [16].

The purpose of this paper is to present some results for on the energy-shell $3p3h$ effects in the response function. This is done in the context of the full ETRPA approach which is therefore reviewed below. The nature of the resulting response function is then confronted to what one obtains using the ESRPA by performing a perturbative expansion of the responses in each case. The possibility of having a three nucleon ejection process is finally analyzed in this framework.

Let us start with the linear response to an external field $\hat{F}$ defined as

$$R(E) = -i \int_{-\infty}^{\infty} \langle\tilde{0}|T\left[\hat{F}^{H\dagger}(t)\hat{F}^{H}(0)\right]|\tilde{0}\rangle e^{iEt}dt, \qquad (1)$$

where $\hat{F}^H(t) \equiv e^{i\hat{H}t}\hat{F}e^{-i\hat{H}t}$, $\hat{H} = \hat{H}_0 + \hat{V}$, with $\hat{H}_0$ and $\hat{V}$ being, respectively, the Hartree-Fock (HF) mean field and the residual interaction. The spectral representation of the response

function, in terms of a set $\{|\nu\rangle\}$ of eigenstates of the hamiltonian $\hat{H}$, reads

$$R(E) = \sum_\nu \left[\frac{\langle\tilde{0}|\hat{F}|\nu\rangle\langle\nu|\hat{F}^\dagger|\tilde{0}\rangle}{E - E_\nu + i\eta} - \frac{\langle\tilde{0}|\hat{F}^\dagger|\nu\rangle\langle\nu|\hat{F}|\tilde{0}\rangle}{E + E_\nu - i\eta} \right], \tag{2}$$

where η is an infinitesimal positive number.

Within the equation of motion method [1], the set $\{|\nu\rangle\}$ is generated as

$$|\nu\rangle = \Omega_\nu^\dagger|\tilde{0}\rangle; \;\; \Omega_\nu^\dagger = \sum_i X_i^\nu C_i^\dagger - \sum_j Y_j^\nu C_j, \tag{3}$$

and

$$\Omega_\nu|\tilde{0}\rangle = 0, \qquad \text{for all } \nu. \tag{4}$$

The operators $C_i^\dagger$ and C_i (with $C_i^\dagger \equiv a_{p_1}^\dagger \cdots a_{p_i}^\dagger a_{h_1} \cdots a_{h_i}$) create and annihilate i particle-hole pairs on the HF vacuum $|0\rangle \equiv |0p0h\rangle$, respectively.

The equation of motion for $\Omega_\nu^\dagger$

$$\langle\tilde{0}| \left[\Omega_\nu, \left[H, \Omega_\mu^\dagger\right]\right] |\tilde{0}\rangle = E_\nu \langle\tilde{0}| \left[\Omega_\nu, \Omega_\mu^\dagger\right] |\tilde{0}\rangle \delta_{\nu,\mu}, \tag{5}$$

where E_ν stands for the excitation energy of the state $|\nu\rangle$, leads to the RPA-like eigenvalue problem

$$\mathcal{A}\mathcal{X}^\nu = E_\nu \mathcal{N}\mathcal{X}^\nu, \tag{6}$$

with

$$\mathcal{A} = \begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix}, \quad \mathcal{X}^\nu = \begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix}, \quad \mathcal{N} = \begin{pmatrix} N & 0 \\ 0 & -N^* \end{pmatrix}. \tag{7}$$

The submatrices A, B and N given by

$$A_{i,j} = \langle\tilde{0}| \left[C_i, \left[H, C_j^\dagger\right]\right] |\tilde{0}\rangle, \;\; B_{i,j} = \langle\tilde{0}| \left[C_i, \left[H, C_j\right]\right] |\tilde{0}\rangle, \;\; N_{i,j} = \langle\tilde{0}| \left[C_i, C_j^\dagger\right] |\tilde{0}\rangle, \tag{8}$$

and using eqs. (2-6) it is possible to write the response in representation independent form as

$$R(E) = \mathcal{F}^\dagger (E\mathcal{N} - \mathcal{A} + i\eta\mathcal{I})^{-1} \mathcal{F}, \tag{9}$$

where $\mathcal{F}$ is defined as

$$\mathcal{F} \equiv \begin{pmatrix} F^A \\ F^B \end{pmatrix}, \text{with} \begin{cases} F_i^A = \langle\tilde{0}| \left[C_i, \hat{F}\right] |\tilde{0}\rangle, \\ F_i^B = F_i^{A*}(\hat{F} \rightarrow \hat{F}^\dagger). \end{cases} \tag{10}$$

After splitting the Hilbert space of $ipih$ states into a P-space that includes only the $1p1h$ states and the Q-space that spans on the rest of the states, the response function can be written as

$$R(E) = \tilde{\mathcal{F}}_P^\dagger(E)\mathcal{G}_P(E)\tilde{\mathcal{F}}_P(E) + \mathcal{F}_Q^\dagger \mathcal{G}_Q(E)\mathcal{F}_Q, \tag{11}$$

where

$$\mathcal{G}_P(E) = \left[E\mathcal{N}_P + i\eta\mathcal{I}_P - \mathcal{A}_P - (\mathcal{A}_{PQ} - \mathcal{N}_{PQ}E)\,\mathcal{G}_Q(E)\,(\mathcal{A}_{QP} - \mathcal{N}_{QP}E)\right]^{-1}, \tag{12}$$

with

$$\mathcal{G}_Q(E) = \left[E\mathcal{N}_Q + i\eta\mathcal{I}_Q - \mathcal{A}_Q\right]^{-1}, \tag{13}$$

and

$$\tilde{\mathcal{F}}_P(E) = \mathcal{F}_P - \mathcal{N}_{PQ}\mathcal{F}_Q + \mathcal{A}_{PQ}\mathcal{G}_Q(E)\mathcal{F}_Q. \tag{14}$$

In standard RPA the state $|\tilde{0}\rangle$ is approximated by the HF ground state and the Q-space is absent, while the so called extended RPA incorporates perturbative ground state $2p2h$ admixtures and a perturbatively suggested truncation of the dynamical matrices and excitation operator. It is obtained by:
i) evaluating the matrix elements (8) and (10) for [11]

$$|\tilde{0}\rangle = c_0|0\rangle + \sum_{2_0} c_{2_0}|2_0\rangle, \tag{15}$$

where

$$c_0 \cong 1 - \frac{1}{2}\sum_{2_0}|c_{2_0}|^2\,, \quad c_{2_0} \cong -\frac{V_{2_0 0}}{E_{2_0}}, \tag{16}$$

$2_0 \equiv (p_1p_2h_1h_2)_0$ represents the $2p2h$ ground state admixtures, E_{2_0} the corresponding unperturbed energy and $V_{2_0 0} \equiv \langle 2_0|V|0\rangle$, and
ii) keeping terms up to second order in $\hat{V}$ for the forward sector within the P space, terms linear in $\hat{V}$ for the backward sector within the P space and for the coupling between the P and Q spaces, and only terms of zeroth order within the Q space. Under these conditions the norm matrix elements read [5])

$$N_{ij} = \delta_{ij} + \Delta N_{ij} \tag{17}$$

where $i \equiv ipih$ and the nonzero ΔN_{ij} are

$$\Delta N_{11'} = \sum_{2_0,2'_0} c^*_{2_0} c_{2'_0} \langle 2_0|\hat{D}_{11'}|2'_0\rangle, \;\; \Delta N_{13} = \sum_{2_0} c^*_{2_0} \langle 1; 2_0|3\rangle, \tag{18}$$

where $\hat{D}_{11'} = \left[\hat{C}_1, \hat{C}^\dagger_{1'}\right] - \delta_{11'}$ and $\langle 1; 2_0|3\rangle$ is the overlap between the $1p1h \otimes (2p2h)_0$ and $3p3h$ final state configurations. (Note that within the quasi-boson approximation $\hat{D}_{11'} \equiv 0$.) The explicit result for the matrix element $\langle 2_0|\hat{D}_{11'}|2'_0\rangle$ is

$$\begin{aligned} \langle (p_1p_2h_1h_2)_0|\hat{D}_{ph,p'h'}|(p'_1p'_2h'_1h'_2)_0\rangle &= -\left[1 + P(h_1,h_2)P(h'_1,h'_2)\right] \\ \times \;\; \Big[\delta_{p,p'}\delta_{h_1,h'}P^-(h,h_2)P^-(p_1,p_2)&\delta_{h'_1,h}\delta_{h_2,h'_2}\delta_{p_2,p'_2}\delta_{p_1,p'_1}\Big] + p \leftrightarrow h, \end{aligned} \tag{19}$$

where $P^-(i,j) \equiv [1 - P(i,j)]$, while the operator $P(i,j)$ exchanges the arguments i and j.

The forward going energy matrix elements are evaluated in the same way and one gets

$$A_{ij} = \delta_{ij}E_j + V_{ij} + \Delta A_{ij}, \tag{20}$$

where $V_{ij} \equiv \langle i|\hat{V}|j\rangle$ and the nonzero matrix elements ΔA_{ij} are:

$$\Delta A_{11'} = \sum_{2_0,2'_0} (E_1 - E_{2_0}) c^*_{2_0} c_{2'_0} \langle 2_0|\hat{D}_{11'}|2'_0\rangle, \;\; \Delta A_{13} = \Delta N_{13}E_3. \tag{21}$$

The one-body matrix elements are:

$$F^A_i = \begin{cases} f_1 + \sum_{1'} \Delta N_{11'} f_{1'} & \text{for } i = 1 \\ \\ \sum_{2_0} c_{2_0} f_{i2_0} & \text{for } i > 1, \end{cases} \tag{22}$$

where

$$f_1 \equiv \langle 1|\hat{F}|0\rangle, \;\; \text{and} \;\; f_{i2_0} \equiv \langle i|\hat{F}|2_0\rangle. \tag{23}$$

Before proceeding it is convenient to introduce the unperturbed Green's function:

$$\mathcal{G}^0(E) \equiv \begin{pmatrix} G^0(E) & 0 \\ 0 & G^{0*}(-E) \end{pmatrix}, \tag{24}$$

where $G^0(E) \equiv [E^+ - A(\hat{H} = \hat{H}_0)]^{-1}$ (with $E^+ \equiv E + i\eta$) and rewrite the perturbed Green function within the space P in the form

$$\mathcal{G}_P(E) = \left[\left(\mathcal{G}^0_P(E)\right)^{-1} - \mathcal{K}_P(E)\right]^{-1}, \tag{25}$$

where

$$\mathcal{K}_P(E) \equiv \mathcal{K}_{11'}(E) = \begin{pmatrix} V_{11'} + \Sigma_{11'}(E) & B_{11'} \\ B^*_{11'} & V^*_{11'} + \Sigma^*_{11'}(-E) \end{pmatrix}, \tag{26}$$

with

$$\Sigma_{11'}(E) = \Delta\Sigma^{(2)}_{11'}(E) + \Delta\Sigma^{(3)}_{11'}(E) + \sum_{\mathsf{i}=2,3} V_{1\mathsf{i}} G^0_{\mathsf{ii}}(E) V_{\mathsf{i}1'}, \tag{27}$$

and

$$\begin{aligned} \Delta\Sigma^{(2)}_{11'}(E) &= \Delta A_{11'} - \Delta N_{11'} E, \\ \Delta\Sigma^{(3)}_{11'}(E) &= -\left(2V_{13} - \Delta N_{13}\left(G^0_{33}(E)\right)^{-1}\right)\Delta N_{31'}. \end{aligned} \tag{28}$$

In the above equations $V_{\mathsf{ij}'}$ stands for the matrix representation of the residual interaction within the $ipih \otimes jpjh$ subspace.

The response function now reads

$$R(E) = \tilde{\mathcal{F}}_{1'}(E)\mathcal{G}_{11'}(E)\tilde{\mathcal{F}}_{1'}(E) + \sum_{\mathsf{i}=2,3} \mathcal{F}^\dagger_{\mathsf{i}} \mathcal{G}^0_{\mathsf{ii}}(E)\mathcal{F}_{\mathsf{i}}, \tag{29}$$

where

$$\tilde{\mathcal{F}}_1(E) \equiv \begin{pmatrix} \tilde{F}^A_1(E) \\ \tilde{F}^B_1(E) \end{pmatrix}, \text{with} \begin{cases} \tilde{F}^A_1(E) = f_1 + \Delta\tilde{F}_1(E), \\ \Delta\tilde{F}_1(E) = \Delta F^{(2)}_1 + \Delta F^{(3)}_1 + \sum_{\mathsf{i}=2,3} V_{1\mathsf{i}} G^0_{\mathsf{ii}}(E) F_{\mathsf{i}} \\ \Delta F^{(2)}_1 = \Delta N_{11'} f_{1'}, \quad \Delta F^{(3)}_1 = -\Delta N_{13} F_3. \end{cases} \tag{30}$$

From the expressions for $\Delta A_{11'}$ and $\Delta N_{11'}$, given by Eqs. (18) and (21) respectively, the matrix elements $\Delta\Sigma^{(2)}_{11'}(E)$ and $\Delta\tilde{F}^{(2)}_1$ can be expressed as:

$$\Delta\Sigma^{(2)}_{11'}(E) = -\sum_{2_0,2'_0} c^*_{2_0} c_{2'_0} \langle 2_0|\hat{D}_{11'}|2'_0\rangle (E - E_1 + E_{2'_0}), \tag{31}$$

$$\Delta\tilde{F}^{(2)}_1 = \sum_{2_0,2'_0} c^*_{2_0} c_{2'_0} \langle 2_0|\hat{D}_{11'}|2'_0\rangle f_{1'}. \tag{32}$$

Moreover, from the relationships

$$V_{13} = -\sum_{2_0} c^*_{2_0} E_{2_0} \langle 1; 2_0|3\rangle; \quad f_{3,2_0} = \sum_1 \langle 3|1; 2_0\rangle f_1, \tag{33}$$

one obtains

$$\Delta\Sigma^{(3)}_{11'} = \sum_{2_0,2'_0} c^*_{2_0} c_{2'_0} \langle 1; 2_0|1'; 2'_0\rangle (E - E_1 + E_{2'_0}), \tag{34}$$

$$\Delta\tilde{F}^{(3)}_1 = -\sum_{2_0,2'_0} c^*_{2_0} c_{2'_0} \langle 1; 2_0|1'; 2'_0\rangle f_{1'}. \tag{35}$$

We can note here that

$$\langle 1; 2_0|1'; 2'_0\rangle = \langle 2_0|(\hat{D}_{11'} + \hat{d}_{11'})|2'_0\rangle, \text{ with } \hat{d}_{11'} \equiv \delta_{11'} + C^\dagger_{1'} C_1, \tag{36}$$

and thus in summary we get:

i) in the ESRPA (where the Q space includes only the $2p2h$ excitations)

$$\Sigma_{11'}(E) = -\sum_{2_0,2'_0} c^*_{2_0} c_{2'_0} \langle 2_0|\hat{D}_{11'}|2'_0\rangle (E - E_1 + E_{2'_0}) + \sum_2 \frac{V_{12}V_{21'}}{E^+ - E_2}, \tag{37}$$

$$\tilde{\mathcal{F}}_1(E) = f_1 + \sum_{2_0,2'_0;1'} c^*_{2_0} c_{2'_0} \langle 2_0|\hat{D}_{11'}|2'_0\rangle f_{1'} + \sum_{2,2_0;} \frac{V_{12} f_{22_0} c_{2_0}}{E^+ - E_2}; \tag{38}$$

ii) in the ETRPA (where the Q space includes both the $2p2h$ and $3p3h$ excitations)

$$\Sigma_{11'}(E) = \sum_{2_0,2'_0} c^*_{2_0} c_{2'_0} \langle 2_0|\hat{d}_{11'}|2'_0\rangle (E - E_1 + E_{2'_0}) + \sum_{i=2,3} \frac{V_{1i}V_{i1'}}{E^+ - E_i}, \tag{39}$$

$$\tilde{\mathcal{F}}_1(E) = f_1 - \sum_{2_0,2'_0} c^*_{2_0} c_{2'_0} \langle 2_0|\hat{d}_{11'}|2'_0\rangle f_{1'} + \sum_{i=2,3;2_0} \frac{V_{1i} f_{i2_0} c_{2_0}}{E^+ - E_i}. \tag{40}$$

The results (37) and (38) are in essence those obtained previously by Arima and collaborators [5, 11] and by the Jülich group [6, 12]. On the other hand, when terms containing the matrix elements $\langle 2_0|\hat{d}_{11'}|2'_0\rangle$ are neglected in eqs. (39) and (40), one finds the results derived in our previous works [16]. [1]

[1] These terms give rise to disconnected graphs, which are nonphysical, as well as to double connected graphs represented in fig. 1d and 1e, respectively. As seen from relations (43) and (47) below, they do not contribute to the response function.

In order to elucidate some of the content of these equations we turn next to a perturbative expansion of the response function and examine the leading corrections to the unperturbed $1p1h$ response $R^0(E) = \sum_1 |f_1|^2/(E^+ - E_1)$. To achieve maximum simplicity we first omit the residual interaction within the $1p1h$ sector and backward contributions, so that to second order the Bethe-Salpeter equation Eq. (25) reads

$$G_{11'}(E) \cong G^0_{11}(E) + G^0_{11}(E)\Sigma_{11'}(E)G^0_{1'1'}(E), \tag{41}$$

which substituted in Eq. (11) leads to the desired approximation for the response function. Within the ESRPA one gets:

$$\begin{aligned} R(E) \cong R^0(E) &+ \sum_{2,2_0,2'_0} c^*_{2_0} \frac{f^*_{22_0} f_{22'_0}}{E^+ - E_2} c_{2'_0} + 2 \sum_{1,2,2_0} \frac{\Re(f^*_1 f_{22_0} c_{2_0})}{E^+ - E_1} \frac{V_{12}}{E^+ - E_2} \\ &+ \sum_{1,1'} \frac{f^*_1}{E^+ - E_1} \left[\sum_{2_0,2'_0} c^*_{2_0} c_{2'_0} \langle 2_0 | \hat{D}_{11'} | 2'_0 \rangle (E - E_1 - E_{2_0}) + \sum_2 \frac{V_{12} V_{21'}}{E^+ - E_2} \right] \frac{f_{1'}}{E^+ - E_{1'}}, \end{aligned} \tag{42}$$

and in the ETRPA:

$$\begin{aligned} R(E) \cong R^0(E) &+ \sum_{i=2,3;2_0,2'_0} c^*_{2_0} \frac{f^*_{i2_0} f_{i2'_0}}{E^+ - E_i} c_{2'_0} + 2 \sum_{i=2,3;1,2_0} \frac{\Re(f^*_1 f_{i2_0} c_{2_0})}{E^+ - E_1} \frac{V_{1i}}{E^+ - E_i} \\ &- \sum_{1,1'} \frac{f^*_1}{E^+ - E_1} \left[\sum_{2_0,2'_0} c^*_{2_0} c_{2'_0} \langle 2_0 | \hat{d}_{11'} | 2'_0 \rangle (E - E_1 - E_{2_0}) - \sum_{i=2,3} \frac{V_{13} V_{31'}}{E^+ - E_i} \right] \frac{f_{1'}}{E^+ - E_{1'}}. \end{aligned} \tag{43}$$

Now the two expressions (42) and (43) can be shown to be equivalent. This results in fact from explicitly performing the sums over $3p3h$ states in Eq. (43). To do that one first rewrites these sums making use of relations (33) and (36) as:

$$\sum_{3,2_0,2'_0} c^*_{2_0} \frac{f^*_{32_0} f_{32'_0}}{E^+ - E_3} c_{2'_0} = \sum_{3,2_0,2'_0} c^*_{2_0} f^*_1 \frac{\langle 2_0 | (\hat{D}_{11'} + \hat{d}_{11'}) | 2'_0 \rangle}{E^+ - E_1 - E_{2_0}} f_{1'} c_{2'_0}, \tag{44}$$

$$2 \sum_{1,3,2_0} \frac{\Re(f^*_1 f_{32_0} c_{2_0})}{E^+ - E_1} \frac{V_{13}}{E^+ - E_3} = -2 \sum_{1,2_0,2'_0} c^*_{2_0} f^*_1 \frac{E_{2_0} \langle 2_0 | (\hat{D}_{11'} + \hat{d}_{11'}) | 2'_0 \rangle}{(E^+ - E_1)(E^+ - E_1 - E_{2_0})} f_{1'} c_{2'_0}. \tag{45}$$

and

$$\sum_{1,1',3} \frac{f_1^*}{E^+ - E_1} \frac{V_{13}V_{31'}}{E^+ - E_3} \frac{f_{1'}}{E^+ - E_{1'}} = \sum_{1,1',2_0,2'_0} \frac{f_1^* c_{2_0}^* E_{2_0} \langle 2_0 | (\hat{D}_{11'} + \hat{d}_{11'}) | 2'_0 \rangle E_{2'_0} f_{1'} c_{2'_0}}{(E^+ - E_1)(E^+ - E_1 - E_{2_0})(E^+ - E_{1'})}, \qquad (46)$$

The result of performing the sum is:

$$\sum_{1,1',2_0,2'_0} \frac{f_1^*}{(E^+ - E_1)} c_{2_0}^* c_{2'_0} \langle 2_0 | (\hat{d}_{11'} + \hat{D}_{11'}) | 2'_0 \rangle (E - E_1 - E_{2_0}) \frac{f_{1'}}{(E^+ - E_{1'})}, \qquad (47)$$

which substituted in Eq. (43) gives the expression (42) also for the ETRPA response.

The cancellation among the $3p3h$ on the energy-shell contributions can be exhibited also making use of the Rayleight-Schrödinger perturbation expansion, i.e.,

$$|\tilde{i}\rangle = |i\rangle + |i\rangle^{(1)} + \cdots \quad \text{and} \quad \tilde{E}_i = E_i + E_i^{(1)} + \cdots, \; i = ipih, \qquad (48)$$

where the perturbed wave functions and energies are indicated by the symbol $\sim$ and the superscript points the order of the correction introduced by the residual interaction $\hat{V}$ on the unperturbed quantities $|i\rangle$ and E_i. The amplitude for the $\hat{F}$- excitation from the correlated ground state to the perturbed $3p3h$ states reads

$$\langle \tilde{3} | \hat{F} | \tilde{0} \rangle = \frac{\langle \tilde{3} | [\hat{H}, \hat{F}] | \tilde{0} \rangle}{\tilde{E}_3 - \tilde{E}_0} = \frac{\langle 3 | [\hat{H}, \hat{F}] | 0 \rangle}{E_3 - E_0} + \mathcal{O}(\hat{V}^2), \qquad (49)$$

with

$$\langle 3 | [\hat{H}, \hat{F}] | 0 \rangle = \sum_1 \langle 3 | \hat{V} | 1 \rangle \langle 1 | \hat{F} | 0 \rangle - \sum_{2_0} \langle 3 | \hat{F} | 2_0 \rangle \langle 2_0 | \hat{V} | 0 \rangle \equiv 0, \qquad (50)$$

where the last equivalence is a direct consequence of the relations (33), i.e., [2]

$$\sum_1 V_{31} f_1 = - \sum_{1,2_0} c_{2_0} E_{2_0} \langle 1; 2_0 | 3 \rangle f_1 = \sum_{2_0} f_{3,2_0} V_{2_0 0} \qquad (51)$$

Thus we see once more that, up to the second order in $\hat{V}$, the $3p3h$ final states do not contribute to the response function and that $|\langle \tilde{3} | \hat{F} | 0 \rangle|^2 \cong \mathcal{O}(\hat{V}^4)$. The Goldstone diagrams for

[2] Note that $\hat{H}_0$ does not contribute since $\langle 3 | [\hat{H}_0, \hat{F}] | 0 \rangle = 0$.

the fourth order $3p3h$ on the mass-shell contributions to the response function are shown in figs. 1e and 1f.

At first glance it might look as if the connected Goldstone diagrams associated with the terms (44), (45) and (46) of the ETRPA response (illustrated in Figs. 1a, 1b and 1c, respectively) should give rise to on the mass-shell $3p3h$ contributions, through the imaginary part of the propagator $(E^+ - E_3)^{-1}$. However, Eq. (47) shows that these contributions in fact cancel out so that the $3p3h$ sector only affects the $1p1h$ excitations by coupling them with the virtual intermediate states $|1; 2_0\rangle$. Thus in spite of including the $3p3h$ propagator in the Green's function, three nucleon ejection does not occur in the leading order processes. The above mentioned diagrams also explain the physical meaning of the fourth term in the expression (42). The cancellation of on shell $3p3h$ contributions results from the destructive interference between amplitudes involving creation of the $3p3h$ state from a ground state correlation and from V_{31} coupling respectively. A similar calculation in which the backward part of Eq. (25) and/or the residual interaction within the $1p1h$ space are kept up to the relevant order leads again to the same result. It is worth stressing that this does not depend on the form of the two-body force used as residual interaction or on the size of single particle space.

References

[1] D.J. Rowe, Nuclear Collective Motion (Methuen, London 1970).

[2] G.F. Bertsch and I. Hamamoto, Phys. Rev. **C26**, 1323 (1982).

[3] C. Yannouleas, M.D. Worzecka and J.J. Griffin, Nucl. Phys. **A397**, 239 (1983).

[4] W.M. Alberico, M. Ericson and A. Molinari, Ann. Phys. (NY) **154**, 356 (1984).

[5] K. Takayanagi, K. Shimizu and A. Arima, Nuc. Phys. **A444**, 436 (1985).

[6] S. Drożdż, V. Klemt, J. Speth and J. Wambach, Phys. Lett. **166B**, 253 (1986).

[7] M.H. Macfarlane, Phys. Lett. **182B**, 265 (1986).

[8] J. Hirsch, A. Mariano, M. Faig and F. Krmpotić, Phys. Lett. **B210** (1988) 55.

[9] S. Adachi and E. Lipparini, Nuc. Phys. **A489**, 445 (1988).

[10] S. Nishizaki, S. Drożdż, J. Wambach and J. Speth, Phys. Lett. **215B**, 231 (1988).

[11] K. Takayanagi, K. Shimizu and A. Arima, Nuc. Phys. **A477**, 205 (1988), **A481**, 313 (1988).

[12] S. Drożdż, S. Nishizaki, J. Speth and J. Wambach, Phys. Rep. **197**, 1 (1990).

[13] W.M. Alberico, A. De Pace, A. Drago and A. Molinari, Preprint DFTT 7/90.

[14] A. Mariano, J. Hirsch and F. Krmpotić, Nuc. Phys. **A518**, 523 (1990).

[15] K. Takayanagi, Phys. Lett. **230B**, 11 (1989); Nuc. Phys. **A510**, 162 (1990); Nuc. Phys. **A516**, 276 (1990); Nuc. Phys. **A522**, 494 (1991).

[16] A. Mariano, E. Bauer, F. Krmpotić, and A.F.R de Toledo Piza, Phys. Lett. **268B**, 332 (1991).

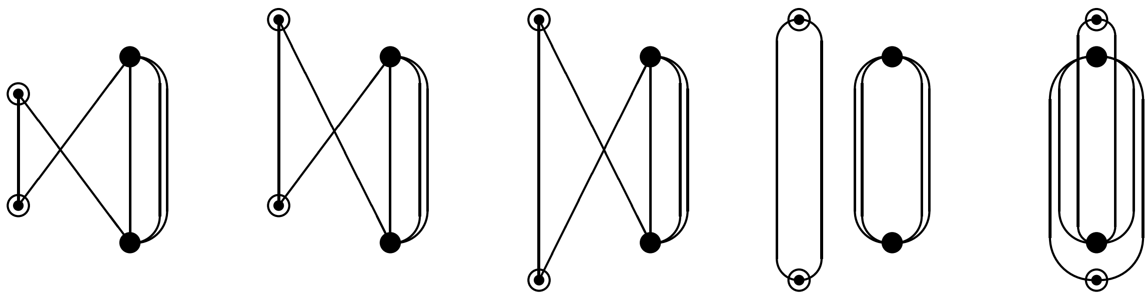
[17] C. Gardee, J. Rapaport, T.N. Taddeucci, C.D. Goodman, C. Foster, D.E. Bainum, C.A. Goulding, M.G. Greenfield, D.J. Horen, and E. Sugarbaker, Nuc. Phys. **A369**, 258 (1981).

[18] J. Rapaport, in AIP Conf. Proc., Vol.**92**, ed. by M.O. Meyer (American Institute of Physics, New York, 1983) p. 365.

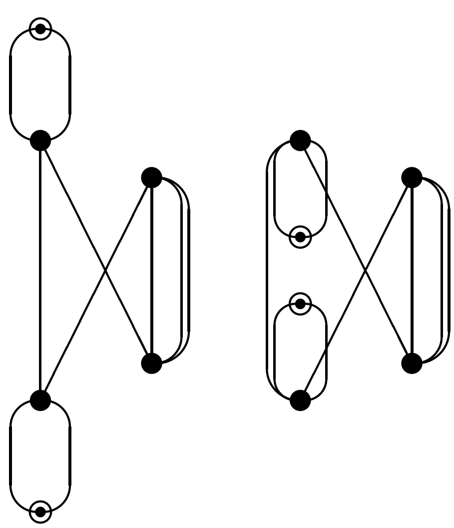
[19] P. Barreau, M. Bernheim, J. Duclos, J.M. Finn, Z. Meziani, J. Morgenstern, J. Mougey, D. Royer, B. Saghai, D. Tarnowski, S. Turck-Chieze, M. Brussel, G.P. Capitani, E. De Sanctis, S. Frullani, F. Garibaldi, D.B. Isabelle, E. Jaus, I. Sick and P.D. Zimmerman, Nuc. Phys. **A402**, 515 (1983).

[20] A. Mariano, Proc. of the XV Workshop on Nuclear Physics, Buenos Aires, Argentina, p. 216 (World Scientific, Singapore 1993).

Figure 1: Graphical representation of the second and fourth order contributions to the response function. The dotted circles ($\odot$) denote the one-body vertices and the filled ones ($\bullet$) indicate the two-body matrix elements. The diagrams (a), (b) and (c) correspond, respectively, to the terms given by eqs. (44), (45) and (46). Second order unlinked and double-linked graphs analogous to the diagram (c) are shown in figures (d) and (e), respectively. The last ones, although contained in eqs. (39) and (40), do not contribute to the response function. Finally, figure (f) illustrates the fourth order on the energy-shell $3p3h$ processes.



(a) (b) (c) (d) (e)



(f)

Figure 1